

Linear algebra and Lattices

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Vectors and matrices

```
? V = [1, 2, 3];
```

```
? W = [4, 5, 6]~;
```

```
? M = [1, 2, 3; 4, 5, 6]
```

```
%3 =
```

```
[1 2 3]
```

```
[4 5 6]
```

```
? V*W
```

```
%17 = 32
```

```
? M*W
```

```
%18 = [32, 77]~
```

```
? U = [1..10]
```

```
%19 = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

Components

? $V[2]$

%20 = 2

? $W[1..2]$

%21 = [4, 5]~

? $M[2,2]$

%22 = 5

? $M[1,]$

%23 = [1, 2, 3]

? $M[, 2]$

%24 = [2, 5]~

? $M[1..2, 1..2]$

%12 =

[1 2]

[4 5]

Constructors

```

? V=vector(10,i,1/i)
%26 = [1,1/2,1/3,1/4,1/5,1/6,1/7,1/8,1/9,1/10]
? W=vectorv(10,i,1/i)
%27 = [1,1/2,1/3,1/4,1/5,1/6,1/7,1/8,1/9,1/10]~
? [1/i | i<-[1..10]]
%28 = [1,1/2,1/3,1/4,1/5,1/6,1/7,1/8,1/9,1/10]
? [1/i | i<-[1..10]]~
%29 = [1,1/2,1/3,1/4,1/5,1/6,1/7,1/8,1/9,1/10]~
? M=matrix(4,4,i,j,i*j)
%30 = [1,2,3,4;2,4,6,8;3,6,9,12;4,8,12,16]
? matdiagonal([1,2,3,4])
%31 = [1,0,0,0;0,2,0,0;0,0,3,0;0,0,0,4]

```

building matrices

Matrices internally are lines of columns.

```
? C1 = [1,2,3]~; C2=[4,5,6]~;
```

```
? concat(C1,C2)
```

```
%33 = [1,2,3,4,5,6]~
```

```
? matconcat([C1,C2])
```

```
[1,4;2,5;3,6]
```

```
? matid(5)
```

```
? matconcat([C1,C2])
```

linear algebra

Linear algebra follows French convention, matrices act on column vectors on the left.

```
? matdet([1,2;3,4]) \\ determinant
```

```
%37 = -2
```

```
? M = [1,2,3;4,5,6];
```

```
? M~ \\ transposition
```

```
%39 = [1,4;2,5;3,6]
```

```
? matsize(M) \\ dimensions
```

```
%40 = [2,3]
```

```
? matrank(M) \\ rank
```

```
%41 = 2
```

```
? K=matker(M) \\ kernel
```

```
%42 = [1;-2;1]
```

```
? M*K
```

```
%43 = [0;0]
```

LLL reduction

```
? V = vector(10, i, random(10^10));  
? M = matconcat([matid(10), V]~);  
? T = qflll(M)  
%46 = [2, 2, 3, 3, -2, -2, 1, 5, -1, 3; ...]  
? B = qflll(M, 3)  
%47 = [2, 2, 3, 3, -2, -2, 1, 5, -1, 3; ...]  
? M*T==B  
%48 = 1  
? Q = M~*M;  
? U = qflllgram(Q)  
%50 = [2, 2, 3, 3, -2, -2, 1, 5, -1, 3; ...]  
? T == U  
%51 = 1
```

Lattices

Example: the Gram matrix of the E8 lattice

```
? E8 = matrix(8, 8, i, j, if(i==1&&j==1, 4, \
      i==j || (i==1 && j<8) || (j==1 && i<8), 2, 1))
? E8==E8~ \\ symmetric
%53 = 1
? matdet(E8) \\ unimodular
%54 = 1
? qfsign(E8) \\ signature
%55 = [8, 0]
? L = qfminim(E8); L[1..2] \\ 240 minimal vectors o
%56 = [240, 2]
? V = L[3][, 1] \\ one minimal vector
%57 = [3, -1, -1, -1, -1, -1, -2, 2]~
? qfeval(E8, V) \\ the norm is 2
%58 = 2
```


Lattices

```
? qfperfection(E8) \\ perfection rank
%59 = 36
? G=qfauto(E8); G[1] \\ number of isometries
%60 = 696729600
? A=G[2][1] \\ one isomorphim
%61 = [0,-1,0,-2,0,-1,1,-1;0,1,0,1,0,0,0,1;1,1,0,1,
? A~*E8*A==E8
%62 = 1
```

Lattices and modular forms

```
? [mf,F,C]=mffromqf(E8);  
? mfparams(F)  
%63 = [1,4,1,y,t-1]  
? mfcoefs(F,10)  
%64 = [1,240,2160,6720,17520,30240,60480,82560,1404  
? mfcoef(F,100003)  
%65 = 240021600648006720  
? L = lfunqf(E8);  
? lfunparams(L)  
%66 = [1,4,[0,1]]  
? lfun(L,0)  
?67 = -1;
```

Exercise

Does the same for the Leech lattice:

- ? `V=concat([vector(23,i,2*i+1),51,145]);`
- ? `K=matkerint(Mat(V));`
- ? `M=matdiagonal(vector(25,i,if(i==25,-1,1)));`
- ? `L24 = K~*M*K;`